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System Identification : Unitree Go2

Akshunn Trivedi, Zachary Lin, Cesar Ramos Chuquiure

The Problem

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Quadrupeds are everywhere around us

Unitree Go2

Boston Dynamics Spot

Source: Respective manufacturers websites

Quadrupeds are everywhere around us

Navigate uneven terrain

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Quadrupeds are everywhere around us

Navigate uneven terrain

Climb stairs

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Quadrupeds are everywhere around us

Navigate uneven terrain

Climb stairs

Do deliveries, perform inspection, do search and rescue, …

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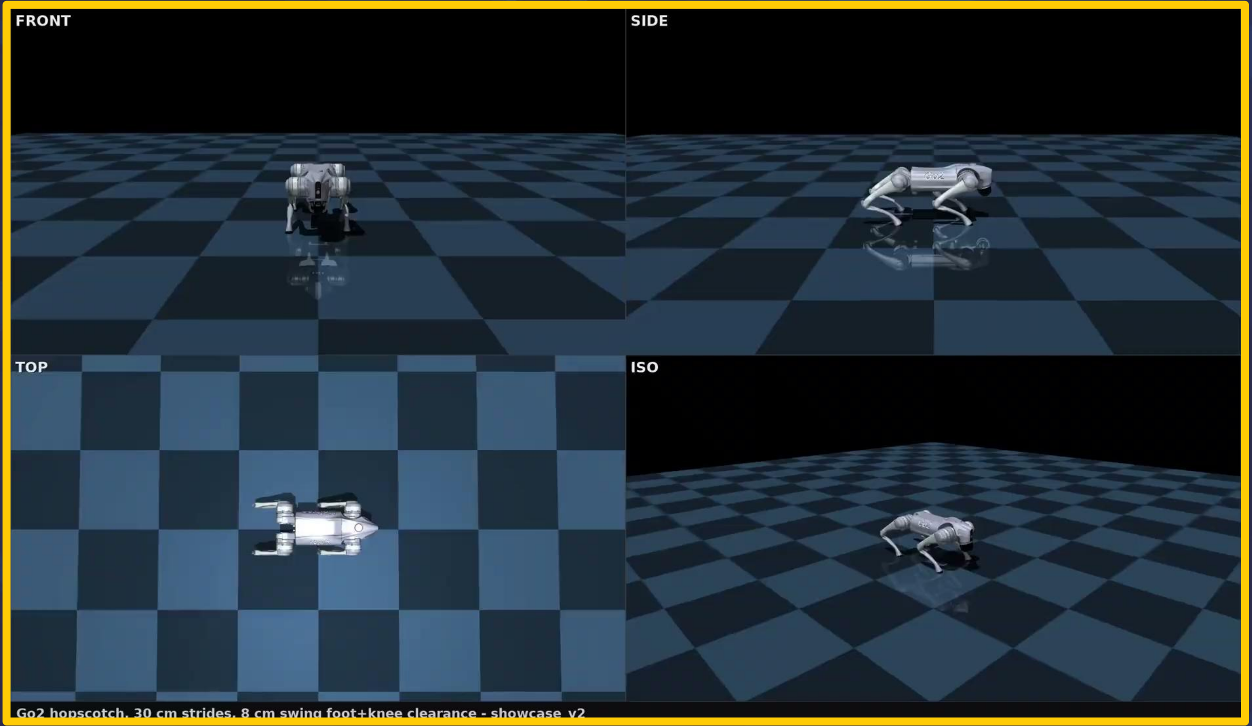
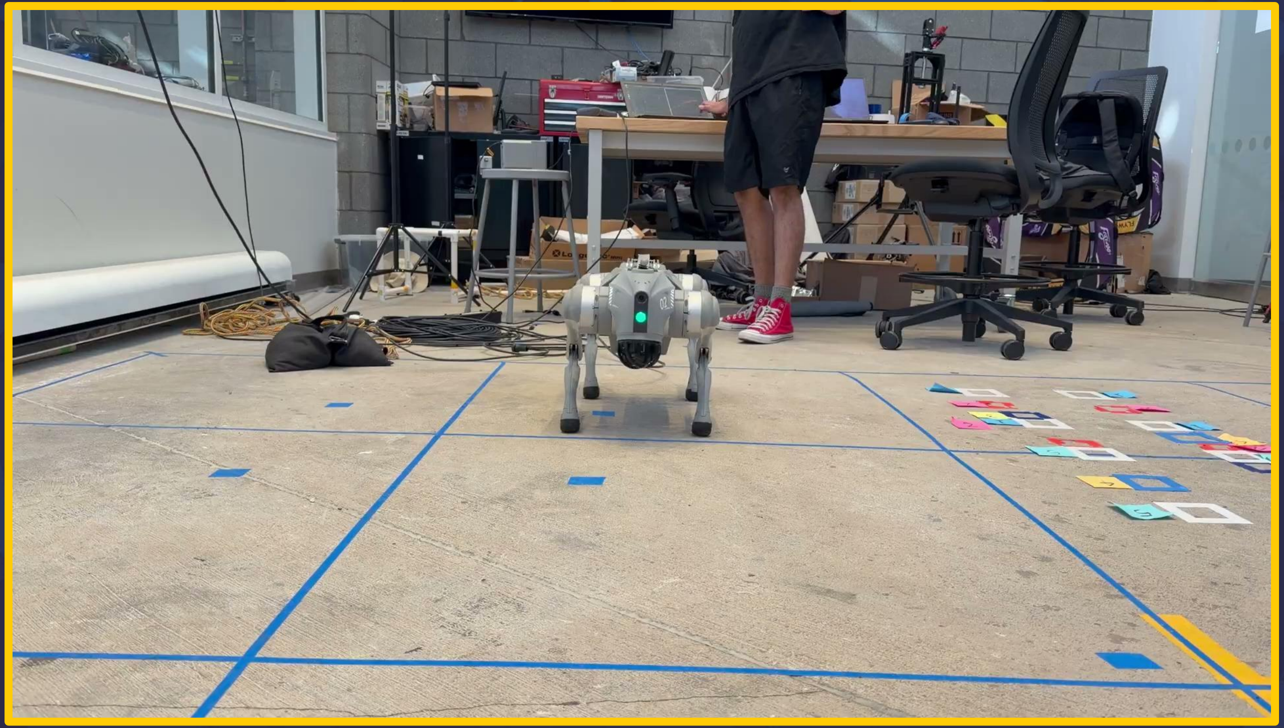
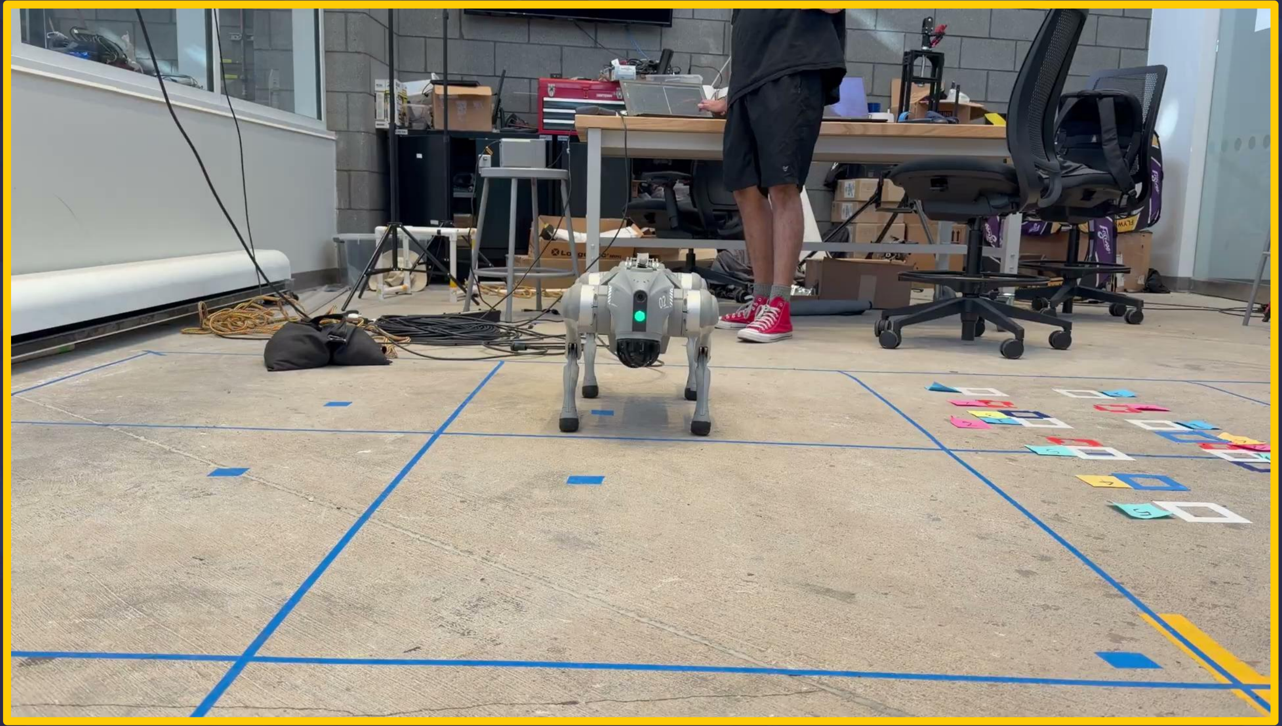
Problem : Do hopscotch on the Go2



Problem : Do hopscotch on the Go2



Problem : Do hopscotch on the Go2 (Sim2Real god strikes, again)



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What might be going wrong?

- Contact/Impact Modelling error
  - Sim has fixed friction, contact is rigid-body, real world differs

What might be going wrong?

- Contact/Impact Modelling error
  - Sim has fixed friction, contact is rigid-body, real world differs
- State estimation errors
  - Flight motion of the hopscotch can introduce drift

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What might be going wrong?

- Incorrect/Unidentified physical parameters
  - URDF parameters are sourced from imprecise CAD models
  - Assembly imperfections
  - Joint motor friction (Coulomb, Viscous) are not captured in CAD models

System Identification 101

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For a single rigid body inertial parameters are defined as:

$$\boldsymbol{\phi}_j = [m, h_x, h_y, h_z, i_{xx}, i_{xy}, i_{xz}, i_{yy}, i_{yz}, i_{zz}]^\top \in \mathbb{R}^{10}$$

where m represents the body mass, h = [hx, hy, hz] = mc represents the first moment of mass, c ∈ R3 being the CoM position in the body frame, and i being elements of moment of inertia matrix expressed in body frame about joint origin

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where m represents the body mass, h = [hx, hy, hz] = mc represents the first moment of inertia, c ∈ R3 being the CoM position in the body frame, and i being elements of moment of inertia matrix expressed in body frame about joint origin

Joint friction parameters are defined as:

$$\theta_{f,j} = [F_{c_j}, F_{v_j}, I_{a_j}, \beta_j]^T,$$

where these components model static friction, viscous friction, transmission inertia, and bias for each joint j

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I=Inertial Parameters

F=Joint Friction Parameters

Trunk (I)

Front legs(I,F)

Rear legs(I,F)

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Dynamics of the Go2

$$\underbrace{H\big(q(t),\theta\big)\,\ddot{q}(t)}_{\text{inertia}} + \underbrace{C\big(q(t),\dot{q}(t),\theta\big)\,\dot{q}(t)}_{\text{Coriolis / centrifugal}} + \underbrace{g\big(q(t),\theta\big)}_{\text{gravity}} + \underbrace{F\big(\dot{q}(t),\ddot{q}(t),\theta\big)}_{\text{friction}} = \underbrace{\tau(t)}_{\text{joint torques}}$$

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Assuming no ground contact forces

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Dynamics of the Go2

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$$F(\dot{q}(t),\ddot{q}(t),\theta) = \underbrace{F_c \circ \operatorname{sign}(\dot{q}(t))}_{\text{Coulomb}} + \underbrace{F_v \circ \dot{q}(t)}_{\text{viscous}} + \underbrace{I_a \circ \ddot{q}(t)}_{\text{transmission inertia}} + \underbrace{\beta}_{\text{bias}}$$

Friction Model

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Both are equivalent

$$\underbrace{W\big(q(t),\dot{q}(t),\ddot{q}(t)\big)}_{\text{depends only on the } \mathit{motion}} \underbrace{\theta}_{\text{the unknowns}} = \tau(t), \qquad W \in \mathbb{R}^{n\times 14n}$$

nonlinear in the motion $\longrightarrow$ **linear in θ**

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Dynamics of the Go2

$$\underbrace{H\big(q(t),\theta\big)\,\ddot{q}(t)}_{\text{inertia}} + \underbrace{C\big(q(t),\dot{q}(t),\theta\big)\,\dot{q}(t)}_{\text{Coriolis / centrifugal}} + \underbrace{g\big(q(t),\theta\big)}_{\text{gravity}} + \underbrace{F\big(\dot{q}(t),\ddot{q}(t),\theta\big)}_{\text{friction}} = \underbrace{\tau(t)}_{\text{joint torques}}$$

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nonlinear in the motion $\longrightarrow$ **linear in $\boldsymbol{\theta}$**

Can be solved by least-squares now!!

System Identification (Legs)

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Provably-Safe, Online System Identification

Bohao Zhang[1], Zichang Zhou[1] and Ram Vasudevan[1]

Provably-Safe, Online System Identification

Bohao Zhang[1], Zichang Zhou[1] and Ram Vasudevan[1]

- Online system identification for the Kinova gen-3 arm unknown payload
    - Provable safe exciting trajectories that can be tracked by an online controller
    - End-effector inertial system identification

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Provably-Safe, Online System Identification

Bohao Zhang[1], Zichang Zhou[1] and Ram Vasudevan[1]

- ~~Online~~ system identification for the ~~Kinova gen-3 arm~~ Unitree Go2 unknown leg inertial params
    - ~~Provable safe~~ exciting trajectories that can be tracked by a ~~online~~ controller
    - ~~End-effector~~ Complete leg inertial system identification

How to do it?

1. Generate an exciting trajectory
2. Record data(q,v,a) using this trajectory on Go2
3. Perform sysid using this recorded data

Exciting trajectory generation

(original) Exciting trajectory generation

$$\min_{k \in K} \quad \texttt{cost}(k) \tag{28}$$

$$\begin{aligned}
& q_j(t;k,\theta) \in [q^-_{j,\text{lim}}, q^+_{j,\text{lim}}] && \forall t \in T, \theta \in [\theta], j \in N_q \\
& \dot q_j(t;k,\theta) \in [\dot q^-_{j,\text{lim}}, \dot q^+_{j,\text{lim}}] && \forall t \in T, \theta \in [\theta], j \in N_q \\
& \tau_j(t;k) \in [\tau^-_{j,\text{lim}}, \tau^+_{j,\text{lim}}] && \forall t \in T, \theta \in [\theta], j \in N_q \\
& \text{FO}_j(q(t;k,\theta)) \bigcap O = \emptyset && \forall t \in T, \theta \in [\theta], j \in N_q
\end{aligned}$$

$$W(k) = \begin{bmatrix} W_e\big(q(t_1,k),\, \dot q(t_1,k),\, \ddot q(t_1,k)\big) \\ \vdots \\ W_e\big(q(t_{N_s},k),\, \dot q(t_{N_s},k),\, \ddot q(t_{N_s},k)\big) \end{bmatrix} \in \mathbb{R}^{N_s n \times 10}$$

$$\text{cost}(k) = \log \kappa_2\big(W(k)\big) = \log \sigma_1 - \log \sigma_{10}$$

Zhang, Zhou & Vasudevan, RSS 2025

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$$\dot{q}_j(t;k,\theta) \in [\dot{q}^-_{j,\text{lim}}, \dot{q}^+_{j,\text{lim}}] \quad \forall t \in T, \theta \in [\theta], j \in N_q$$

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can't be zero!

Reference trajectory: Parametrized as finite fourier series

$$\ddot{q}_i(t,k) = a_{i,0} + \sum_{j=1}^{d} \left(a_{i,j} \cos(j\omega t) + b_{i,j} \sin(j\omega t) \right)$$

$k = \{a_{i,0},\, a_{i,j},\, b_{i,j}\} \in \mathbb{R}^{(2d+1)n}$, $\dot{q}$, q by exact integration from $q(0) = q_0$, $\dot{q}(0) = \dot{q}_0$

Zhang, Zhou & Vasudevan, RSS 2025

But can we just directly port this??

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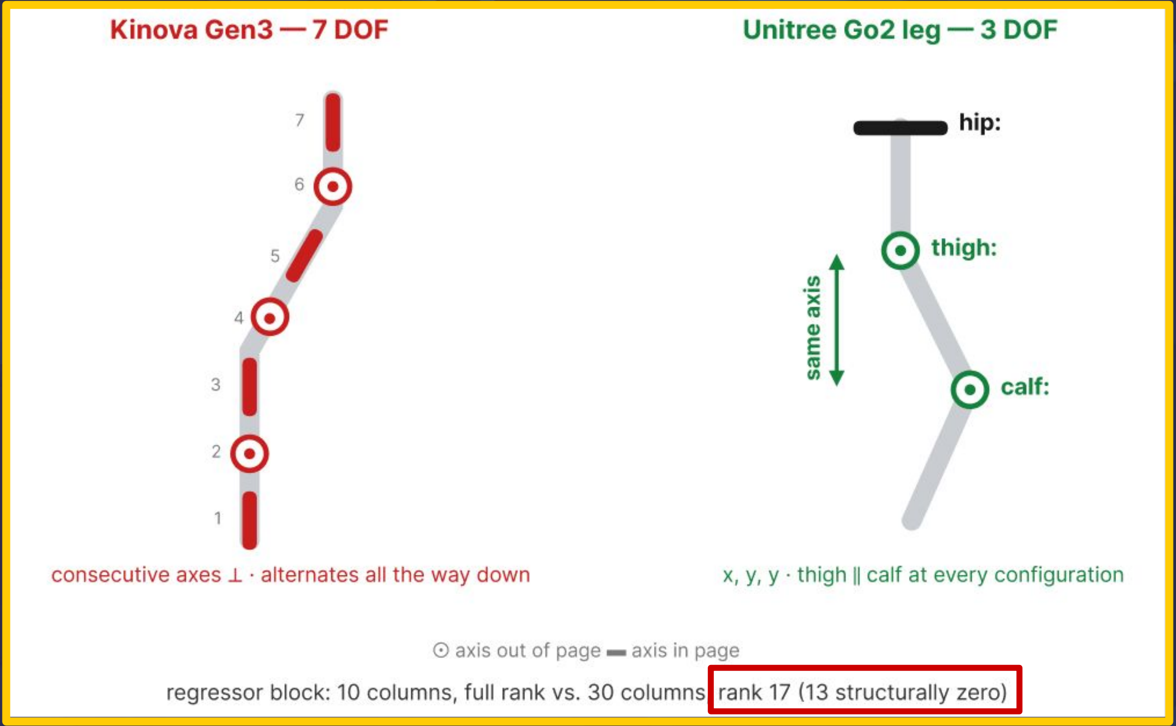

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(original vs ours) Exciting trajectory generation

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$$\texttt{cost}(k) = \log \kappa_2(W(k)) = \log\sigma_1 - \log\sigma_{10}$$

will be zero
for go2

$$\min_{k\in K} \quad \texttt{cost}(k) \tag{28}$$
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$$W(k) = \begin{bmatrix} W_{\mathrm{leg}}(q(t_1,k), \dot q(t_1,k), \ddot q(t_1,k)) \\ \vdots \\ W_{\mathrm{leg}}(q(t_{N_s},k), \dot q(t_{N_s},k), \ddot q(t_{N_s},k)) \end{bmatrix} \in \mathbb{R}^{N_s n\times 30}$$
$$W(k) = U\Sigma V^\top, \quad r = \left|\{ i : \sigma_i \ge \varepsilon\sigma_1 \}\right|, \quad W_{\mathrm{id}}(k) = W(k)\,[v_1 \cdots v_r]$$
$$\texttt{cost}(k) = \log\kappa_2(W_{\mathrm{id}}(k)) = \log\sigma_1 - \log\sigma_r$$

not zero

Exciting trajectory generation: Result

4X Speed

System Identification (Friction)

Remember,

$$F\left(\dot{q}(t), \ddot{q}(t), \theta\right) = \underbrace{F_c \circ \text{sign}\left(\dot{q}(t)\right)}_{\text{Coulomb}} + \underbrace{F_v \circ \dot{q}(t)}_{\text{viscous}} + \underbrace{I_a \circ \ddot{q}(t)}_{\text{transmission inertia}} + \underbrace{\beta}_{\text{bias}}$$

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Using that we can model a general optimization problem

$$\min_{z = [\,F_c \,|\, F_v \,|\, I_a \,|\, \beta\,]} \sum_{i=1}^{N} \tfrac{1}{2} \left\| \hat\tau_i(z) - \tau_i \right\|^2$$

Takes care of gravity,inertia and coriolis terms

$$\text{where} \quad \hat\tau_i(z) = \underbrace{\mathrm{RNEA}\big(q_i, \dot q_i, \ddot q_i;\, \varphi_{\mathrm{URDF}}\big)}_{\text{fixed, independent of } z} + \underbrace{F_c \circ \operatorname{sign}(\dot q_i) + F_v \circ \dot q_i + I_a \circ \ddot q_i + \beta}_{\text{friction model}}$$

$$\text{s.t.} \quad F_c,\ F_v,\ I_a \in [0,\, 50]^n, \qquad \beta \in [-50,\, 50]^n$$

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System Identification (Friction)

Simple procedure:
1) Run a reference trajectory(like a sinusoid with multiple frequencies)
2) Regress over the above equation
3) Repeat the process on a held out trajectory and measure the error to quantify the result as valid.

System Identification (Friction): Results

Joint	F_c [N m]	F_v [N m s/rad]	β [N m]	I_a [10^{-3} kg m^2]
FR hip	0.1461	0.0282	*-0.1302*	*0.399*
FR thigh	0.1217	0.0407	*+0.0890*	4.031
FR calf	1.4840	0.0282	−0.1124	18.549

grey = not identifiable from this trajectory

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grey = not identifiable from this trajectory

(maybe) The acc values were very small compared to thigh and calf, and hence unidentified

System Identification (Inertial)

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Remember,

$$\underbrace{W\big(q(t), \dot{q}(t), \ddot{q}(t)\big)}_{\text{depends only on the } motion} \underbrace{\theta}_{\text{the unknowns}} = \tau(t), \qquad W \in \mathbb{R}^{n \times 14n}$$

nonlinear in the motion $\longrightarrow$ **linear in $\boldsymbol{\theta}$**

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Can get really noisy!

System Identification (Inertial)

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nonlinear in the motion $\longrightarrow$ **linear in $\boldsymbol{\theta}$**

Instead using the law,

$$\underbrace{F = m\,a}_{\text{needs acceleration}} \qquad \longrightarrow \qquad \underbrace{\int_{t_1}^{t_2} F\,dt = m\,(v_2 - v_1)}_{\text{needs only two velocities}}$$

Zhang, Zhou & Vasudevan, RSS 2025

System Identification (Inertial)

it can be derived: (trust me)

$$\underbrace{\left[W_m\big(q(t_2),\dot q(t_2)\big) - W_m\big(q(t_1),\dot q(t_1)\big) - \int_{t_1}^{t_2}\big(W_{C^\top\dot q} - W_g\big)\,dt\right]}_{A} \qquad \theta \qquad =$$

$$\underbrace{\int_{t_1}^{t_2}\big(\tau - F_c \circ \mathrm{sign}(\dot q) - F_v \circ \dot q - \beta\big)dt \;-\; I_a \circ \big(\dot q(t_2) - \dot q(t_1)\big)}_{b}$$

only q, $\dot q$, and integrals of measured torque — no $\ddot q$

Can be solved by least-squares now!!

Zhang, Zhou & Vasudevan, RSS 2025

System Identification (Inertial)

Finally, to make sure obtained parameters are physically consistent, Log-Cholesky parametrization is introduced

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$$\theta_{\mathrm{ip},j} = P(\eta),$$

where the *log-Cholesky parameters* $\eta \in \mathbb{R}^{10}$ are defined

$$\eta = \begin{bmatrix} \alpha & d_1 & d_2 & d_3 & s_{12} & s_{13} & s_{23} & s_{14} & s_{24} & s_{34} \end{bmatrix}$$

and the mapping $P : \mathbb{R}^{10} \to \mathbb{R}^{10}$ is given by

$$P(\eta) = e^{2\alpha} \begin{bmatrix} s_{14}^2 + s_{24}^2 + s_{34}^2 + 1 \\ s_{14} e^{d_1} \\ s_{14} s_{12} + s_{24} e^{d_2} \\ s_{14} s_{13} + s_{24} s_{23} + s_{34} e^{d_3} \\ s_{12}^2 + s_{13}^2 + s_{23}^2 + e^{2d_2} + e^{2d_3} \\ s_{13}^2 + s_{23}^2 + e^{2d_1} + e^{2d_3} \\ s_{12}^2 + e^{2d_1} + e^{2d_2} \\ -s_{12} e^{d_1} \\ -s_{12} s_{13} - s_{23} e^{d_2} \\ -s_{13} e^{d_1} \end{bmatrix}.$$

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Optimize in eta space

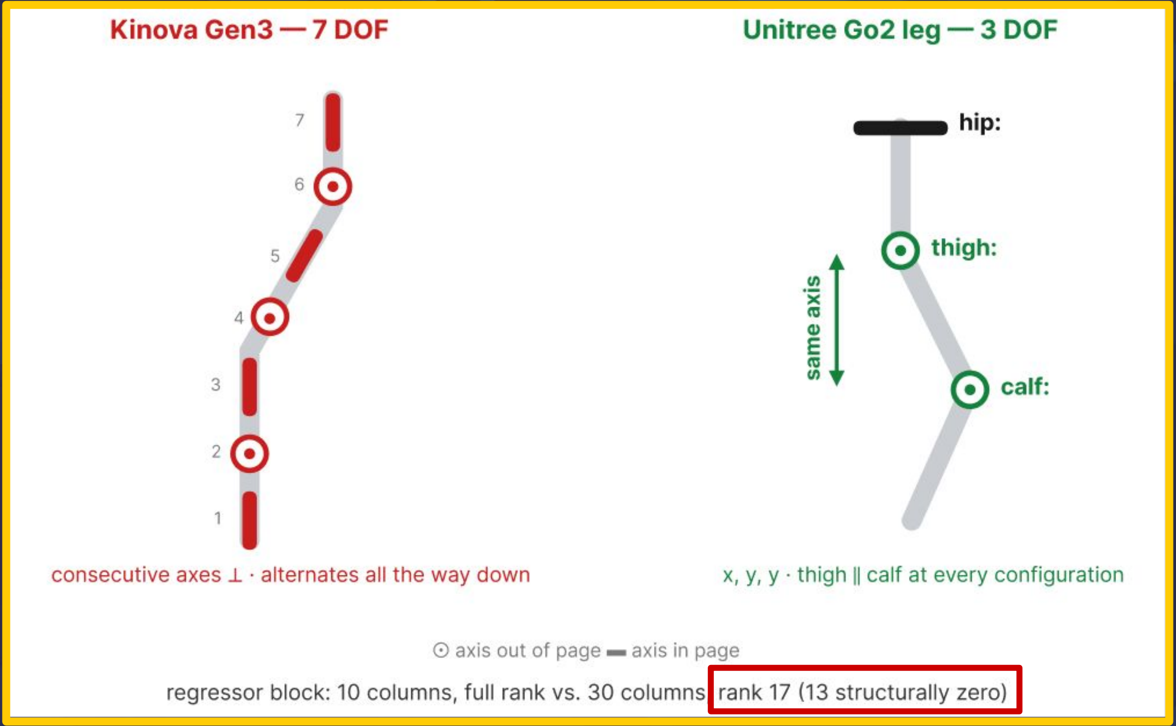
System Identification (Inertial)

But can we just directly port this??

System Identification (Inertial)

But can we just directly port this??
geometrical degeneracy strikes again

System Identification (Inertial)



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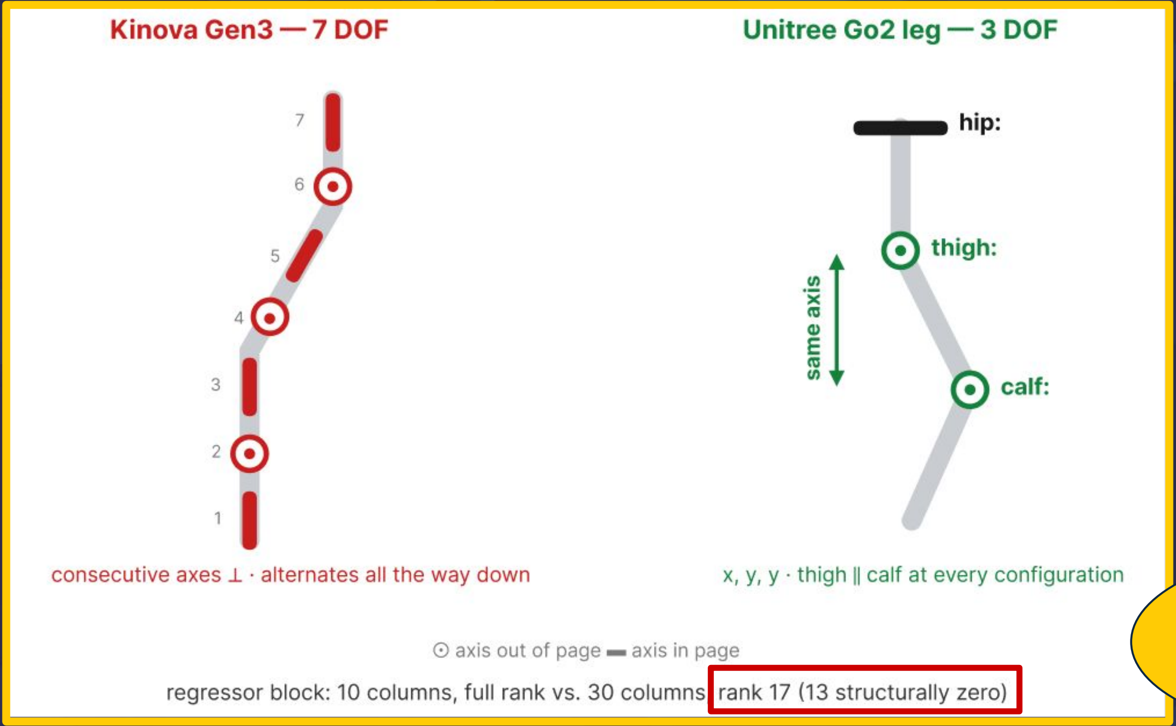
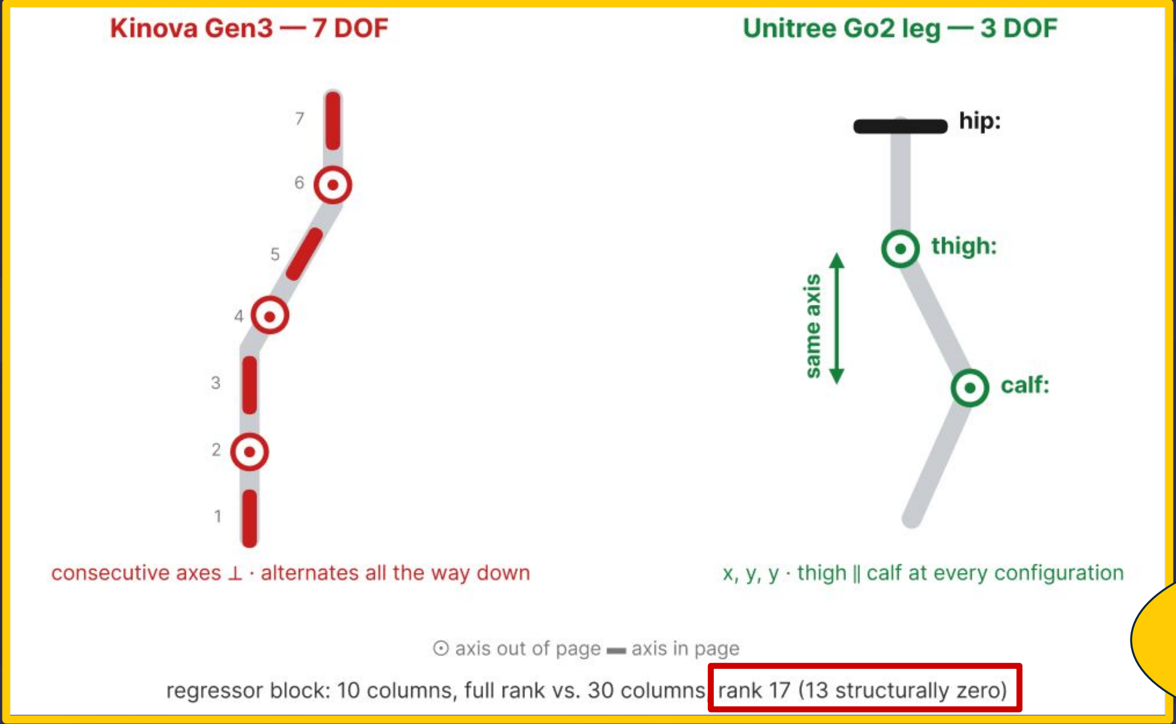


Hessian becomes singular in optimization calculation, non unique solutions

System Identification (Inertial)



Cannot repeat the trick used for exciting traj since objective different

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(Partial) Solution

Original Objective

$$\varphi(z) = \Big[\underbrace{\varphi_{\text{URDF}}}_{\text{links 1..n-1, frozen}} \; ; \; \underbrace{\theta(z)}_{\text{last link}} \Big], \qquad z \in \mathbb{R}^{10}$$

$$\min_{z \in \mathbb{R}^{10}} \; \sum_i \tfrac{1}{2} \left\| A_i \, \varphi(z) - b_i \right\|^2$$

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Updated Objective

$$\varphi(z) = \Big[\underbrace{\theta(z_1) \;;\; \theta(z_2) \;;\; \theta(z_3)}_{\text{every link identified}} \Big], \qquad z \in \mathbb{R}^{30}$$

$$\min_{z \in \mathbb{R}^{30}} \sum_i \tfrac{1}{2} \left\| A_i\, \varphi(z) - b_i \right\|^2 + \tfrac{1}{2} \lambda \left\| \varphi(z) - \varphi_{\text{URDF}} \right\|^2$$

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Pushes the unidentifiable parameters to URDF values; best guess

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small value, .001

Pushes the unidentifiable parameters to URDF values; best guess

System Identification (Inertial): Results

System Identification (Inertial): Results

Link	Parameter	URDF	Identified
Hip	m [kg]	0.678	3.229
	I_{xx} $[10^{-3}\,\mathrm{kg\,m^2}]$	0.483	146.29
	I_{yy}	0.904	137.50
	I_{zz}	0.618	8.79
Thigh	m	1.152	1.272
	I_{xx}	7.645	14.22
	I_{yy}	7.048	60.69
	I_{zz}	1.619	71.26
Calf	m	0.194	0.108
	I_{xx}	4.941	131.20
	I_{yy}	4.966	23.41
	I_{zz}	0.047	140.14

inertias in $10^{-3}\,\mathrm{kg\,m^2}$ · calf row is calf + foot lumped

System Identification (Inertial): Results

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Hip seems basically unidentifiable

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How good/bad these results are tho?

System Identification (Inertial): Validation

1. Design a reference trajectory different from the exciting one for validation.

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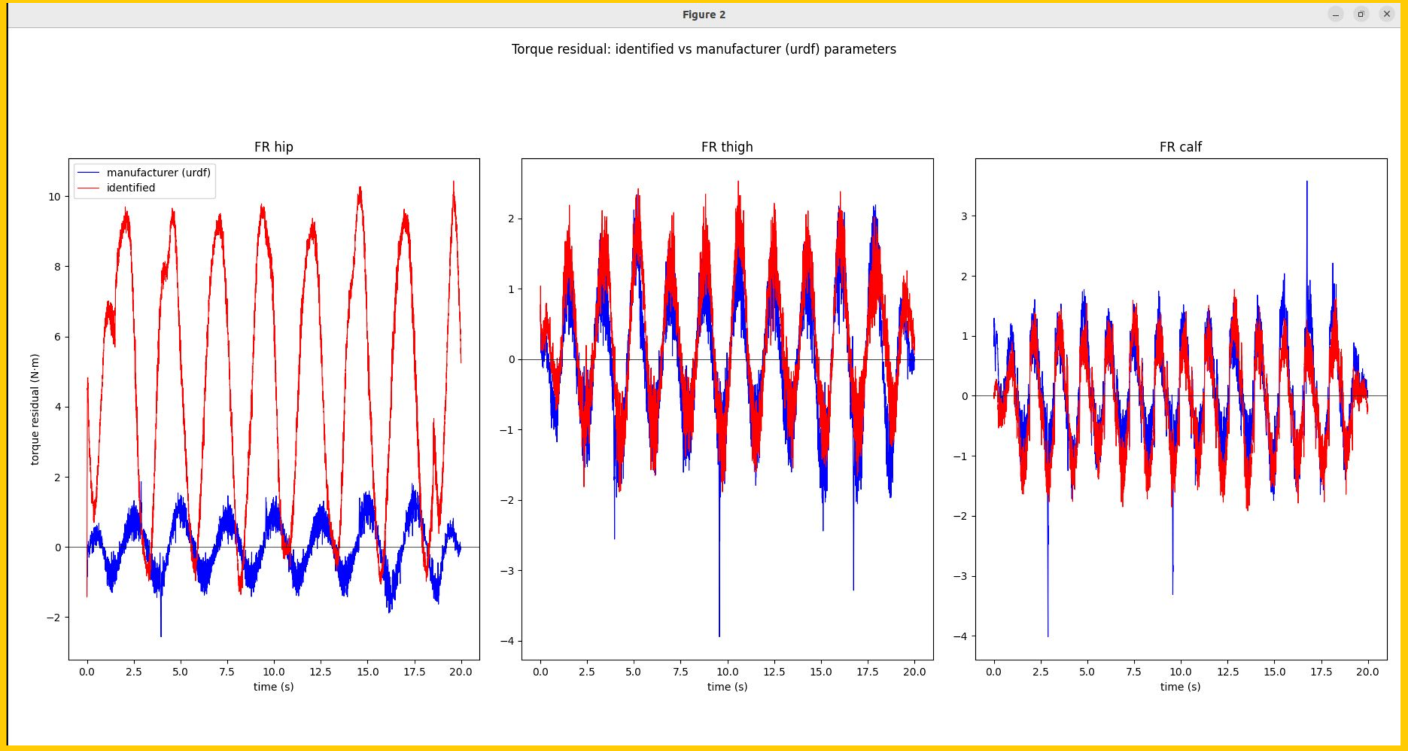
System Identification (Inertial): Validation

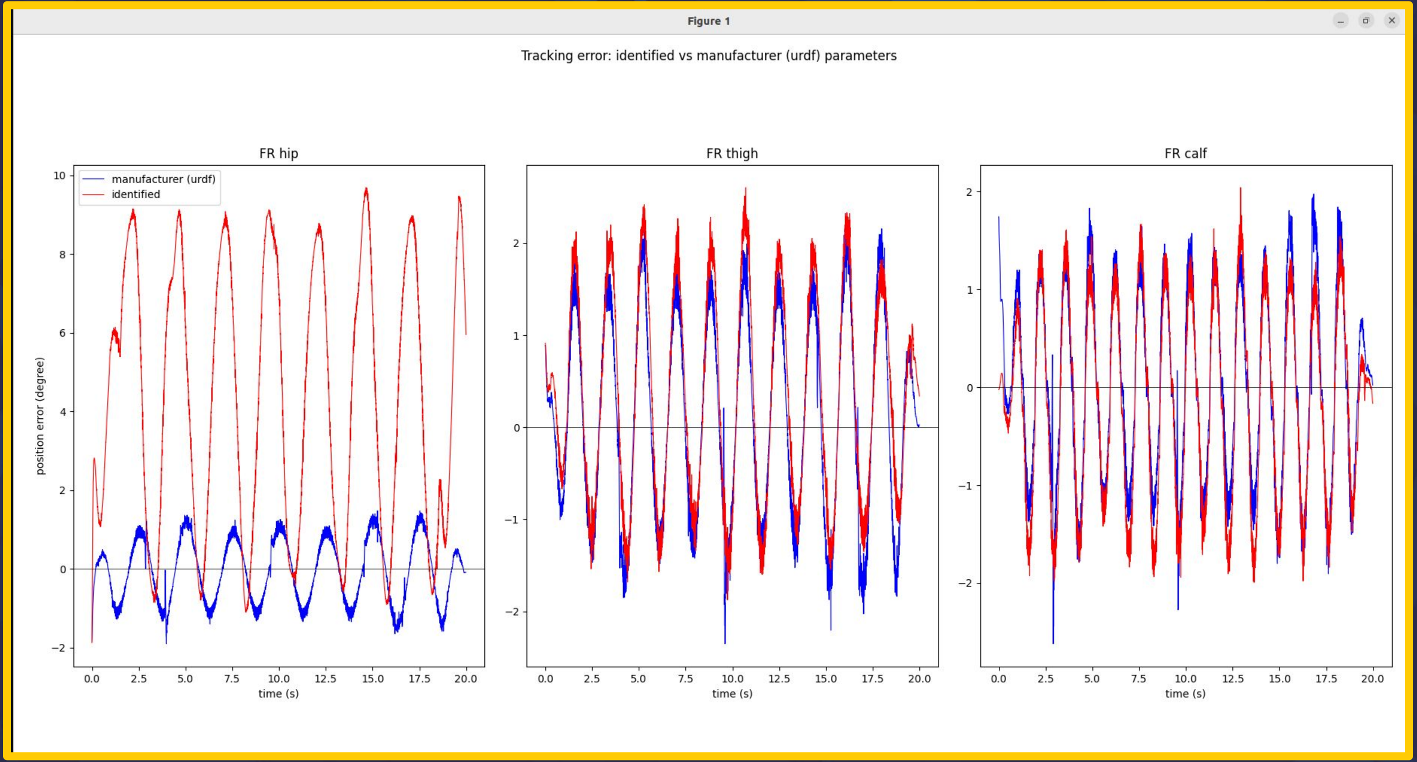
1. Design a reference trajectory different from the exciting one for validation
2. Track this trajectory using a feedforward+feedback controller, and measure the feedback torque. Higher feedback torque means model is bad, lower means it is good.
3. Robot base is pinned.

System Identification (Inertial): Validation Results

System Identification (Inertial): Validation Results



System Identification (Inertial): Validation Results



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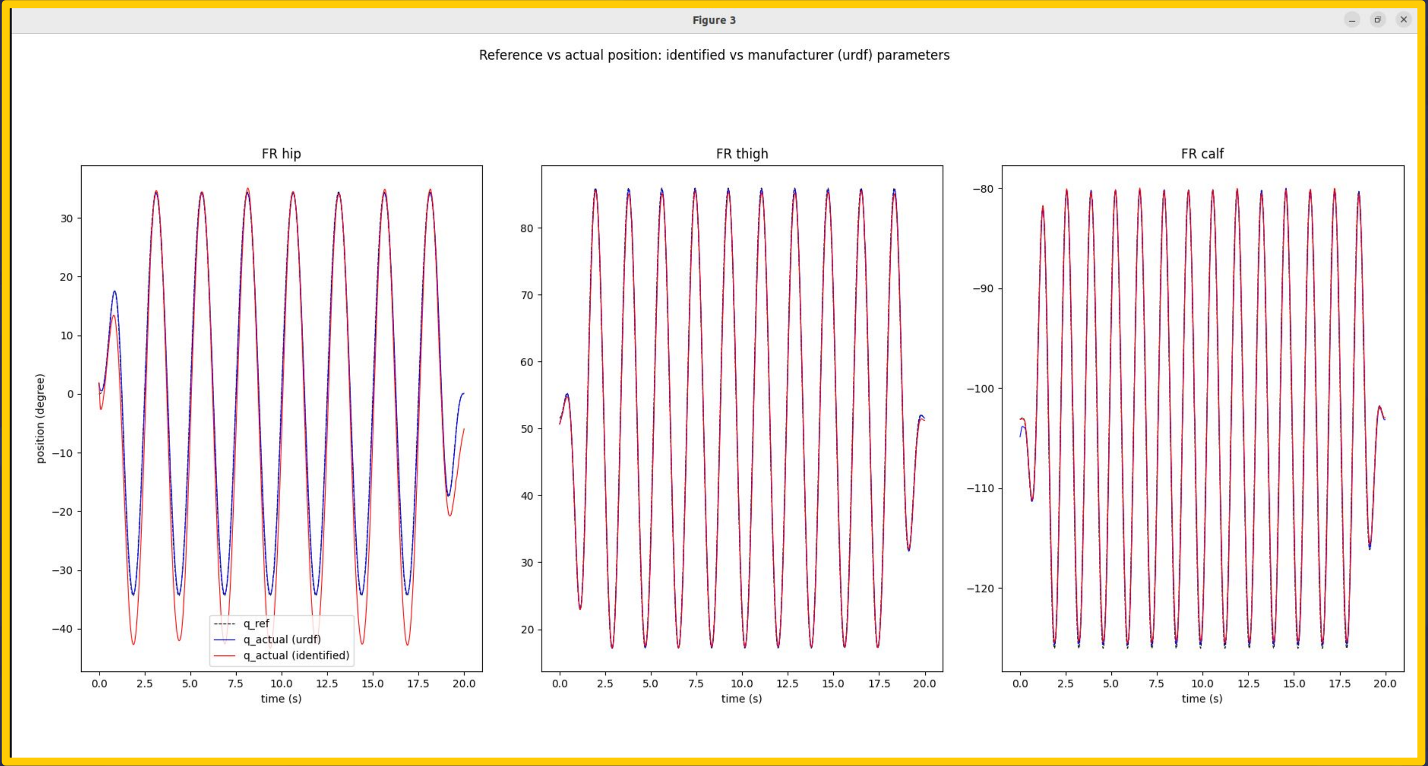

System Identification (Inertial): Validation Results

1. Our estimated values are marginally worse than URDF values.
2. Hip is specially bad
   a. Exciting it is tougher compared to other joints since connected to base
3. For pinned free leg motion, it seems however the model mismatch does not make a ton of difference.

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System Identification (Inertial): Validation Results

1. The trunk is still unaccounted for (about 50% of the entire body by mass)
2. This validation does not prove the model mismatch is low enough for highly dynamic, free floating base motions like hopscotch.
3. (Current work) Need a different validation and sysid procedure (with free floating base motions) to account for hip and trunk.

System Identification (Legs+Trunk)

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The problematic saga of contact

If the issue is pinned trunk does not excite the trunk(obviously) and the hip(from the previous results), we should do sysid on a dynamic trajectory like jumping,trotting etc. Right?

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$$\underbrace{H\left(q(t),\theta\right)\ddot{q}(t)}_{\text{inertia}} + \underbrace{C\left(q(t),\dot{q}(t),\theta\right)\dot{q}(t)}_{\text{Coriolis / centrifugal}} + \underbrace{g\left(q(t),\theta\right)}_{\text{gravity}} + \underbrace{F\left(\dot{q}(t),\ddot{q}(t),\theta\right)}_{\text{friction}} = \underbrace{\tau(t)}_{\text{joint torques}}$$

this holds only for non feet-contact motions.

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The problematic saga of contact

In case of ground reaction forces(contact) the below equation becomes from this

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leg, fixed base: $H(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + g(q) + F \;=\; \tau$

$$\underbrace{M(q)\,\dot{v} + n(q,v)}_{\text{same rigid-body dynamics}} \quad = \quad \underbrace{S^{\top}\tau}_{\text{6 rows unactuated}} \quad + \quad \underbrace{J_c^{\top}\lambda}_{\text{contact forces: unknown}}$$

$$S = \left[\mathbf{0}_{12\times 6} \quad \boldsymbol{I}_{12}\right], \qquad v \in \mathbb{R}^{18}\ (6\ \text{base} + 12\ \text{joints}), \qquad \tau \in \mathbb{R}^{12}, \qquad \lambda \in \mathbb{R}^{3n_c}$$

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need to calculate contact forces now

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The problematic saga of contact

1. Measure it via force plates

The problematic saga of contact

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The problematic saga of contact

1. Measure it via force plates

2. Try to solve it yourself.

3. Try to remove them from the dynamics.

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The problematic saga of contact

Physically-Consistent Parameter Identification of Robots in Contact

Shahram Khorshidi[1], Murad Dawood[1], Benno Nederkorn[2,3], Maren Bennewitz[1], Majid Khadiv[3]

Khorshidi et al., ICRA 2025

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The problematic saga of contact : solution

leg, fixed base: $H(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + g(q) + F \;=\; \tau$

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The problematic saga of contact : solution

$$P \;=\; \mathbf{1}_m - J_c^{\dagger} J_c, \qquad m = n + 6 \qquad \text{orthogonal projector onto } \operatorname{null}(J_c)$$

$$P = P^{\top}, \qquad P^2 = P, \qquad \operatorname{rank}(P) = m - \operatorname{rank}(J_c)$$

$$P\,J_c^{\top} \;=\; \mathbf{0} \qquad \text{whatever } \lambda \text{ is, } P J_c^{\top} \lambda \text{ vanishes}$$

Khorshidi et al., ICRA 2025

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$$P\big(M(q)\dot{v} + n(q,v)\big) \;=\; P\,S^{\top}\tau \qquad \text{motion dynamics — no } \lambda$$

$$(\mathbf{1}_m - P)\big(M(q)\dot{v} + n(q,v)\big) \;=\; (\mathbf{1}_m - P)S^{\top}\tau \;+\; J_c^{\top}\lambda \qquad \text{constraint dynamics — discarded}$$

Khorshidi et al., ICRA 2025

ROBOTICS

The problematic saga of contact : solution

$$Y(q, v, \dot{v})\,\varphi \;=\; S^\top \tau + J_c^\top \lambda$$

$$P\,Y(q, v, \dot{v})\,\varphi \;=\; P\,S^\top \tau \qquad \Big|\ \text{linear least squares in } \varphi$$

Khorshidi et al., ICRA 2025

The problematic saga of contact : solution

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can solve using
least squares
again

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Future Plan:

1. Design exciting trajectories for Go2 to be able to identify most of the trunk and leg physical dynamic parameters
2. Validate and conduct system identification by regressing over the null-space projected dynamics